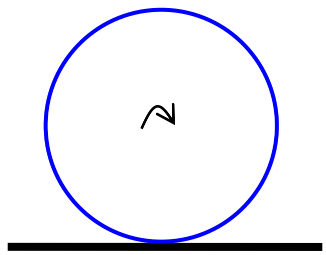
Teacher notes
Topic A

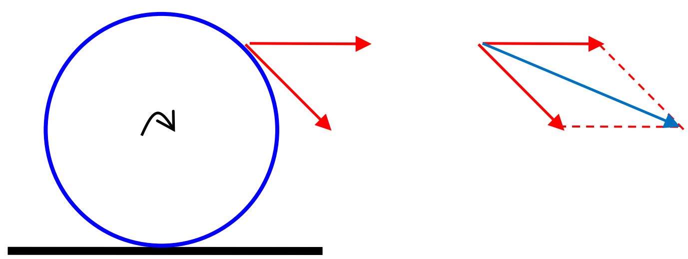
A simple puzzle on the velocities of points on a ring

A ring of radius *R* rolls without slipping on a horizontal road with angular speed ω.



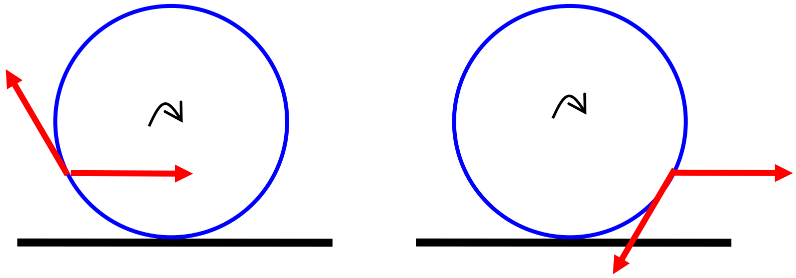
Are there any points on the ring where the speed is equal to the speed of the center of mass of the ring?

A point on the ring has two velocities: a horizontal velocity of magnitude $v_{CM} = \omega R$ due to the motion of the center of mass and a velocity $\vec{v}_T$ tangential to the ring of magnitude also ωR due to the rotation of the ring. These two components are equal in magnitude because the ring rolls without slipping.

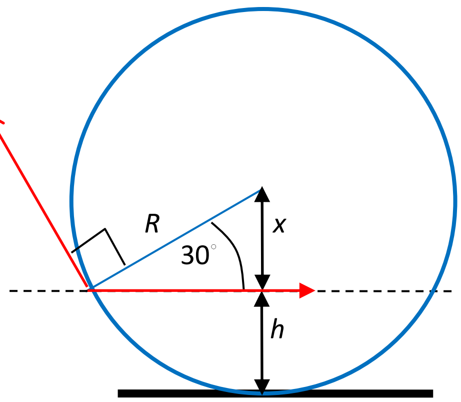


The diagram above shows the two velocities for a particular point on the ring. It is clear, that for this point, the resultant speed will not equal $v_{CM} = \omega R$. We need to add two vectors of equal magnitude such that the sum of the two has the same magnitude as each of the two vectors. We know that this happens when the angle between the vectors is 120°.

This leads to two positions:



It is straightforward to show that the height of each of the two positions from the ground is $\frac{R}{2}$:



$x = R\sin 30^\circ = \frac{R}{2}$, so the height is $h = R - \frac{R}{2} = \frac{R}{2}$. Similarly for the other position.

In general, the speed of an arbitrary point on the ring is given by (use the cosine rule or dot product)

$$v = \sqrt{(\omega R)^2 + (\omega R)^2 + 2(\omega R)^2 \cos\theta}$$

where θ is the angle between the two velocities.

If $\theta = 120^\circ$ we get $v = \sqrt{(\omega R)^2 + (\omega R)^2 + 2(\omega R)^2 \times (-\frac{1}{2})} = \sqrt{(\omega R)^2} = \omega R$ as expected.

The maximum speed is $v_{max} = \sqrt{(\omega R)^2 + (\omega R)^2 + 2(\omega R)^2 \times 1} = 2\omega R$ (at the top of the ring) and the minimum speed is $v_{min} = \sqrt{(\omega R)^2 + (\omega R)^2 + 2(\omega R)^2 \times (-1)} = 0$ (at the bottom of the ring).

We understand these results intuitively since at the top point $\vec{v}_{CM}$ and $\vec{v}_T$ are parallel so the magnitudes add and at the bottom point they are anti-parallel so they subtract to zero.